Math 53 Final 12/18/03 Hutchings

11. Calculate $\int_C F \cdot dr$ where C is the space curve $r(t) = \langle t, \sin t, \sin t \rangle$, $0 \le t \le \pi$, and $F = \langle x, \sin(\sin y), \cos(\cos z) \rangle$.

$x(t) = t \quad y(t) = \sin t \quad z(t) = \sin t$

$F(x(t), y(t), z(t)) = \langle t, \sin(\sin(\sin t)), \cos(\cos(\sin t)) \rangle$

$= t\,i + \sin(\sin(\sin t))\,j + \cos(\cos(\sin t))\,k$

$r'(t) = 1i + \cos t\, j + \cos t\, k$

$\int_C F \cdot dr = \int_a^b F(x(t), y(t), z(t)) \cdot r'(t)\,dt$

$= \int_0^\pi \left(t\,i + \sin(\sin(\sin t))\,j + \cos(\cos(\sin t))\,k\right) \cdot (1\,i + \cos t\, j + \cos t\, k)\,dt$

$= \int_0^\pi [t + \cos t \sin(\sin(\sin t)) + \cos t \cos(\cos(\sin t))]\,dt$

$= \int_0^\pi t\,dt + \int_0^\pi \cos t \sin(\sin(\sin t))\,dt + \int_0^\pi \cos t \cos(\cos(\sin t))\,dt$

- $\int_0^\pi t\,dt = \frac{t^2}{2}\Big]_0^\pi = \frac{(\pi)^2}{2} - \frac{(0)^2}{2} = \frac{\pi^2}{2}$ *

- $\int_0^\pi \cos t \sin(\sin(\sin t))\,dt$ $\quad u = \sin t \quad 0 \le t \le \pi$ $\quad du = \cos t\,dt \quad 0 \le u \le 0$

  $\int_0^0 \sin(\sin u)\,du = 0$ *

- $\int_0^\pi \cos t \cos(\cos(\sin t))\,dt$ $\quad u = \sin t \quad 0 \le t \le \pi$ $\quad du = \cos t\,dt \quad 0 \le u \le 0$

  $\int_0^0 \cos(\cos(u))\,du = 0$ *

$\frac{\pi^2}{2} + 0 + 0 = \boxed{\frac{\pi^2}{2}}$